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Design requirements for optical fibres in small radii bends

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An experimental method is presented for determining the allowable design stress is small radii bends of an optical fibre in service. The design stress must allow adequate static fatigue lifetime at high reliability. The experimental method and data analysis presented here have been used to describe successfully the static fatigue behaviour in a wide range of environments in tests of up to 48 months duration.

1. Introduction
A commonly used design criterion in the optical fibre industry is that fibres should not be used in applications where the long-term applied stress on the fibre is greater than 33% of the proof test stress. This criterion was developed for optical glass fibres used in long-length telecommunications cables. Unfortunately, applying this design criterion to fibres in small radii bending applications, results in minimum allowable bend radii that are unacceptable, given the need for increasingly compact fibre optic devices. For example, this proof stress criterion allows for only a minimum bend radius of about 37 mm for a 125 μm diameter fibre proof tested at 0.35 GPa.

It is the purpose of this paper to demonstrate not only that the proof test stress design criterion is unnecessarily conservative but that the proof test stress generally has little, if any, effect on the long-term performance of optical fibres in small radii bending. A design method applicable to optical glass fibres in small radii bending will be presented with particular emphasis on the necessary experimental data. Based on this analysis, it will be shown that a strength, not a proof test stress, criterion should be used for optical glass fibres in a bending geometry.

2. Experimental procedure
The optical glass fibres used in this study were Corning CPC multimode fibres with acrylate coatings. These fibres were purchased at various times over a 2 year period. The strength of the fibres was measured under ambient conditions in both tension and two-point bending. In tension the fibre had a gauge length of 0.5 m, and was gripped top and bottom with capstan grips fitted to a universal testing machine. The strain rate in tension was 40% min^{-1}. The two-point bending apparatus was similar to that used previously [1, 2]. Essentially, a bent loop of fibre was constrained by grooves between two face plates, and then the face plates are brought together by a computer-controlled stepper motor that stopped when the fracture was sensed by an acoustic detector. The failure stress, S, is calculated from [2]

$$S = 1.198E\frac{2r}{D - d} \qquad (1)$$

where E is the elastic modulus, r the fibre radius, D the face plate separation at fracture, and d the overall fibre diameter (including any coating material). In the bending test the strain rate varies with plate separation, but was approximately 40% min^{-1}. The elastic modulus of the fibre was taken to be 70.3 $(1 + 3r/\rho)$ GPa [3]. To minimize fatigue effects, the strength of the fibres was also measured in two-point bending by first vacuum drying the fibres at room temperature at $< 10^{-5}$ torr (1 torr $= 1.333 \times 10^2$ Pa) for 15 h, and then removing the fibre from vacuum and breaking it in less than 20 s.

A two-point bending geometry was also employed to determine the static fatigue behaviour of the fibres, as described previously [4]. The bending fixture consisted of a block of anodized aluminium 2.5 cm high with 20 precisely reamed identical holes. The ends of the fibres were pulled through each hole until the fibre bends were inside the holes. By constructing several fixtures with a range of hole sizes, applied stresses were systematically varied from about 1.10–3.70 GPa. Each fixture with its 20 specimens was submerged in a Pyrex tank with deionized water whose temperature was carefully regulated to ± 0.2 °C. Deionized water was continuously dripped into each tank, which was equipped with an overflow spout. The pH and conductivity of the water in each tank were monitored regularly. The pH remained relatively constant at about 4–6, and the conductivity at 1–2 $(\mu\Omega\,\text{cm})^{-1}$. The rate of addition of deionized water was sufficient to replace the loss due to evaporation and to turn over the water in the tank every 12 h. A layer of hollow polypropylene balls was floated on the surface of the water to reduce evaporation, and a polycarbonate lid over the tank reduced contamination by dust. Fibre breaks were measured optically by connecting the fibres to an array of 20 light-emitting diodes and bundling them together at a large area silicon photodetector. A computer and data acquisition system checked for fibre continuity by firing the LEDs sequentially, launching light into each fibre, and detecting the optical signal at the photodiode.

There are several important advantages to the two-point bending technique for testing optical glass fibres that are to be used in bending. First is the ease with which fibres can be tested, which facilitates the testing of many specimens simultaneously, especially when several testing tanks are used. Second is that two-point bending closely mimics the geometry of the fibre in actual use. Because the static fatigue behaviour [4–6] of optical glass fibres is a complex function of fibre coatings, test geometry (applied stress and gauge length), and environment (temperature, humidity, chemical environment, pH), it is essential that long-term studies be conducted under conditions that closely simulate those of actual use.

3. Results and discussion

General proof test levels range from 0.35–1.38 GPa, and are significantly lower than strengths of short-gauge length fibres measured either in tension or two-point bending, which can have strengths greater than 5 GPa. Thus, proof testing can eliminate the low-strength processing-related flaws, but has an insignificant effect on the high-strength intrinsic flaws. This can most readily be seen by comparing two-point bending and tensile strength data to the proof test stress. To do this the tensile strength data will be normalized to the two-point bending loading configuration. Assuming that a two-parameter Weibull distribution [7] can represent the tension and two-point bending data, then

$$F_t = 1 - \exp\left[-K_t A_t \left(\frac{S}{\sigma_{0t}}\right)^{m_t}\right] \tag{2}$$

$$F_b = 1 - \exp\left[-K_b A_b \left(\frac{S}{\sigma_{0b}}\right)^{m_b}\right] \tag{3}$$

where subscripts t and b designate tension and two-point bending, respectively, F is the failure probability, K a dimensionless stress state constant, A the surface area under test, and m and σ_0 are the Weibull modulus and scaling parameter, respectively. The constant K for tension is simply unity, and for two-point bending it is [2]

$$K_b = \frac{1}{\pi^{1/2}} \frac{\Gamma[(m_b + 1)/2]}{\Gamma[(m_b + 2)/2]} \tag{4}$$

Combining Equations 2 and 3 yields the failure probability of the tensile test normalized to the two-point bending test

$$F_t = 1 - (1 - F_b)^{\alpha} \tag{5}$$

where

$$\alpha = \frac{A_t (S/\sigma_{0t})^{m_t}}{K_b A_b (S/\sigma_{0t})^{m_b}} \tag{6}$$

Fig. 1 compares the ambient, two-point bending strength of the commercial, acrylate-coated fibre to that determined in tension where failure probability is normalized relative to the two-point bend geometry. Also shown is an estimate of the normalized failure probability of the proof test using Equation 5 and assuming that the proof test stress is tensile and that failure occurs every 10 km. There are several important conclusions that can be drawn for Fig. 1. First is that the flaw population eliminated by the proof test occurs at failure probability levels of 10^{-8} or less. Thus, if one is concerned with failure probabilities of 10^{-5} or higher, proof testing will not affect this higher strength population. Second, the importance of having tensile strength data to supplement the two-point bending data is emphasized because it is the tensile data that extends the strength measurements down to failure probabilities important in design (10^{-4}–10^{-5}).

These conclusions are further dramatized by the static fatigue lifetimes in Table I for this commercial, acrylate-coated optical fibre. This table clearly shows that time to failure does not correlate with proof stress. The difference in failure times observed between the fibres proof tested at 0.35 and 1.38 GPa is thought to be due to small processing variations between the two sets of fibres. Further support that proof testing does not affect the high-strength flaw population is given by Mitsunaga *et al.* [8] (compare Figs 7 and 8 of [8]).

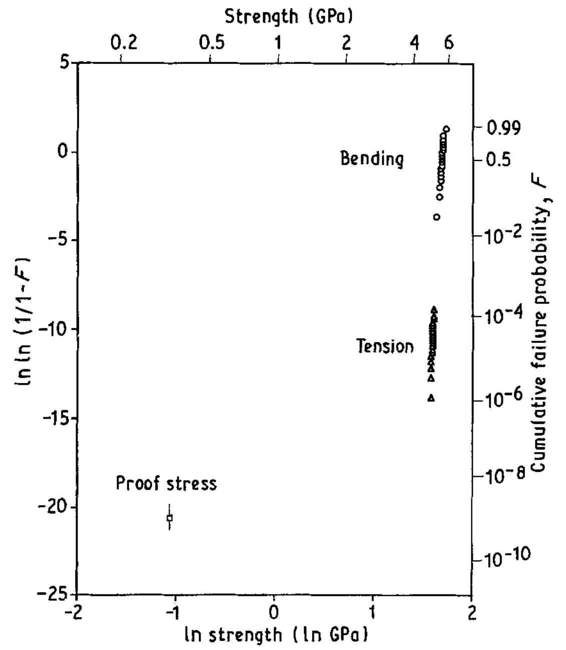

Figure 1 Comparison of two-point bending and tensile strengths of a commercial, acrylate-coated fibre, compared to proof stress.

TABLE I Comparison of two-point bending static fatigue lifetime of commercial fibre purchased with two levels of proof stress. Tests were in 80 °C water immersion at a stress of 3.49 GPa

Proof stress (GPa)	Mean time to failure (s)	Number of specimens
0.35	29 100 ± 10 300	180
1.38	1 700 ± 1 380	150

To use optical fibres reliably in small radii bending, we propose that a design method based on fracture mechanics principles be used in place of the proof test criterion. Static fatigue (failure under constant stress) has been measured extensively in high-silica optical fibres [9, 10]. For fibres that do not show a fatigue transition, failure time, t_f, has been found to obey the fracture mechanics relationship [7, 11]

$$t_f = BS^{n-2}\sigma_a^{-n} \quad (7)$$

where B and n are fatigue constants, S is the inert strength, i.e. the strength measured in the absence of fatigue, and σ_a is the applied stress. Equation 7 shows that failure time increases for higher strength fibres and decreases as applied stress increases. Failure probability is incorporated in Equation 7 by substituting in the appropriate Weibull inert strength distribution, which at the failure probabilities of interest should be given by the inert tensile strength Weibull parameters. Thus, after some algebraic manipulation

$$\ln t_f = \ln B + \frac{n-2}{m_t} \times \left(\ln\ln\frac{1}{1-F} + m_t \ln\sigma_{0t} - \ln KA\right) - n\ln\sigma_a \quad (8)$$

where now K and A refer to the fibre loading geometry in service, and the subscript t refers to tension. It should be noted that Equation 7 can be derived from fracture mechanics principles by assuming the slow growth of a sharp crack to a size sufficient to cause catastrophic failure [7, 11]. In support of this fracture mechanics model, recent research [12] using scanning tunnelling microscopy has shown that atomically sharp cracks exist on high-strength fibres and it is believed that it is these cracks that grow to failure.

From Equation 8 it can be seen that failure predictions depend on the Weibull inert strength parameters and the fatigue constants n and B. For the failure probabilities of interest in design, the appropriate Weibull parameters should be measured in tension; however, the inert strength in tension is experimentally difficult to measure. In bending the inert strength can be measured by removing a specimen from vacuum and quickly testing it (within 10–20 s), Fig. 2. This technique measures inert strengths which are > 14 GPa, the same value as that measured in liquid nitrogen [11, 13, 14]. In tension it is not possible to test a specimen this quickly, and therefore the tensile strength Weibull parameters have to be determined indirectly. Three sets of data are required: the ambient strength in tension and in bending, and the inert strength in bending. The Weibull parameters for inert strength in tension may be calculated from the following expressions [7]

$$\left(\frac{\bar{S}_{t,i}}{\bar{S}_{b,i}}\right)^{(n-2)/(n+1)} = \frac{\bar{S}_{t,a}}{\bar{S}_{b,a}} \quad (9)$$

$$m_{t,i} = m_{t,a}\left(\frac{n-2}{n+1}\right) \quad (10)$$

where $\bar{S}$ is the Weibull median strength and m is the slope, and n is now the value appropriate for the ambient environment. The subscripts t and b refer to tension and bending, and the subscripts a and i refer to ambient and inert testing conditions. The Weibull scaling parameter, $\sigma_{0t,i}$, is calculated from Equation 2, knowing $S_{t,i}$ and $m_{t,i}$. Based on the data in Figs 1 and 2, and using a measured ambient n value of 32, $\sigma_{0t,i}$ = 11.58 GPa and $m_{t,i}$ = 123.

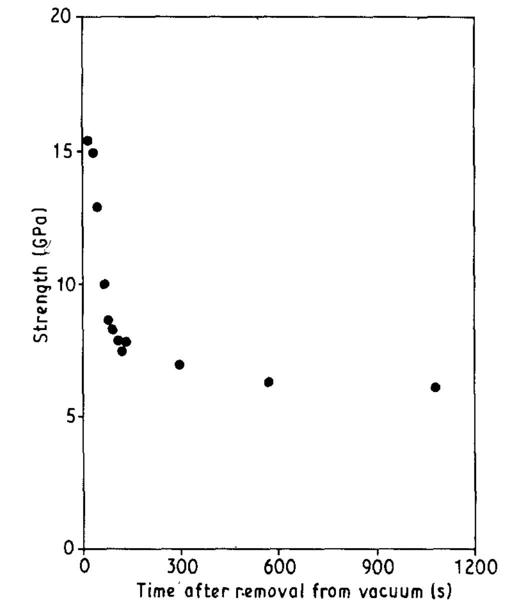

Figure 2 Two-point bending strength of fibre dried at $< 10^{-5}$ torr for 15 h, plotted as a function of time after removal from vacuum.

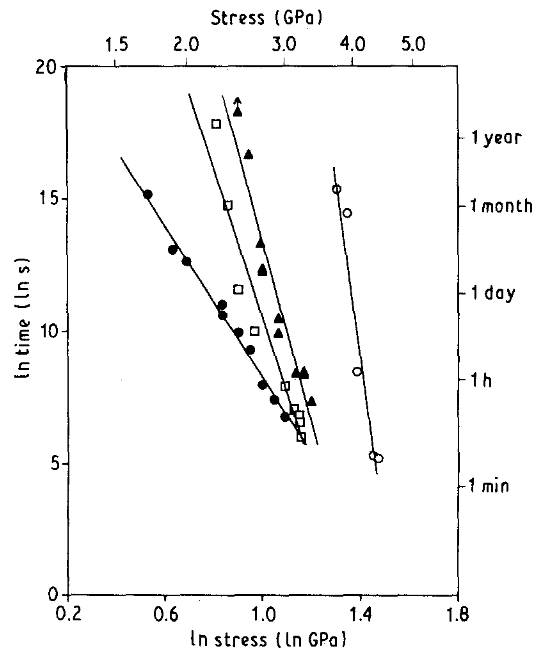

Figure 3 Static fatigue of a commercial, acrylate-coated fibre in moist air at 80 °C. Data taken in two-point bending. The arrow refers to a test still in progress. Relative humidity: (○) 0.1%, (▲) 30%, (□) 60%, (●) 90%.

The fatigue constants n and B appropriate to the service environment can be obtained from the two-point bend fatigue data shown in Fig. 3. Note that in these tests no transition in the fatigue behaviour was observed. From Equation 7 the slope of the data in Fig. 3 gives n and the intercept gives B, knowing the two-point bend strength of the samples. Analysis of the data in Fig. 3 for 90% r.h. gives $n = 14.3$ and $B = 5.23 \times 10^{-5}$ MPas, using the median inert tensile strength of 12.38 GPa (calculated from Equation 9).

A complication in applying Equation 8 for a bending geometry is that the applied stress is not distributed uniformly over the fibre. Therefore, it is convenient to divide the fibre into small finite elements so that the stress in each element can be considered constant ($K = 1$). The total reliability ($1 - F$) is then obtained by taking the product of the reliabilities of each element [15].

As an example of the use of Equation 8, consider the case of a fibre with a circular bend with a radius of curvature 5.75 mm and an arc of 90°. For an acceptable failure probability of say 10^{-4}, the lifetime can be calculated as follows. To good approximation, only one element ($\pm 10°$ from the maximum tensile strength axis along the top of the fibre) needs to be taken. For this element then $K = 1$ and

$$A = \left(\frac{20}{360}\right) 2\pi r \left(\frac{90}{360}\right)(2\pi(\rho + r)) \quad (11)$$

where r is the fibre radius (62.5 μm) and ρ is the radius of curvature (5.75 mm in this example). The applied stress in this element is

$$\sigma_a = \sigma_{max} = \frac{Er}{\rho} \quad (12)$$

This gives for σ_{max} a value of 0.789 GPa. Using the fatigue parameters from Fig. 3 and the tensile strength Weibull parameters in Fig. 1, the lifetime for 90% r.h. and 80 °C is calculated to be 1000 years.

Because lifetime varies with strength to the $n - 2$ power, this predicted lifetime is quite sensitive to fibre strength. The fibre in Fig. 3 has a median ambient tensile strength in 0.5 m gauge lengths of 4.92 GPa, corresponding to an inert strength of 12.38 GPa. If the median tensile strength were 5.20 GPa, corresponding to an inert strength of 13.35 GPa, the predicted lifetime would be 2370 years. The lifetime is also very sensitive to the applied stress, varying with the $-n$ power, as well as to relative humidity, which influences the value of n.

In practice there is a required service life and reliability, and the bend radius must be designed to meet these requirements. Table II gives an example of allowable bend radii as a function of relative humidity and initial fibre strength. The calculations are for a fibre with the fatigue susceptibility observed in Fig. 3, and a tight tensile strength distribution represented by a Weibull slope of 154, the value observed in Fig. 1. The calculations use the same inert bending strength as in the above example. The table shows the bend radius allowable for a 125 μm diameter fibre subjected to one 90° circular bend, if that fibre is to have a 1000 year lifetime at a reliability of 0.9999 ($F = 10^{-4}$). These calculations illustrate the sensitivity of the failure predictions to the Weibull strength parameters. The need for high-quality optical fibres (high strength coupled with a tight strength distribution) is evident. They also show that long lifetimes can be expected even for rather tight bend radii. The largest bend radius in Table II corresponds to an applied stress of 0.73 GPa, which is 2.1 times the proof stress of 0.35 GPa.

TABLE II Design radius which will have a 1000 year life at a reliability of 0.9999. Radius is for a 125 μm fibre having one 90° circular bend. The temperature is 80°C continuous. The allowable bend radius is given as a function of fibre strength and relative humidity

$\bar{S}_{t,a}$ (GPa)	$m_{t,a}$	Bend radius (mm) at r.h. (%)		
		30	60	90
4.60	154	2.84	3.32	6.04
4.92	154	2.66	3.11	5.67
4.92	75	2.68	3.14	5.74
4.92	50	2.70	3.16	5.81
4.92	25	2.76	3.25	6.03
5.20	154	2.52	2.94	5.39

The example calculations given above lead to the conclusion that a strength criterion, instead of a proof test criterion, should be stipulated to the optical fibre supplier to ensure that the fibre can be reliably used in the intended bending application. For the above example, such a criterion should be that the Weibull parameters, as measured on short gauge length (0.5 m) tensile specimens, must be $m > 154$ and median strength > 4.92 GPa. This would allow a bend radius of 5.75 mm for use at 80 °C and 90% r.h., with a lifetime estimate of 1000 years at a reliability of 0.9999.

4. Conclusions

An experimental method and data analysis have been developed which allow the prediction of static fatigue lifetime. This method can be used to determine the allowable design stress on the fibre. The data required are the fatigue resistance (time to failure versus applied stress) and the inert strength. In applications where the applied stress results from bending of the fibre, an important design consideration is the tensile strength of fibre in gauge lengths of about 0.5 m, as this strength corresponds to the strength of bent sections of fibre at low failure probability ($F = 10^{-4}$–10^{-5}). Proof stress is not an appropriate design criterion for fibre in bending applications, as proof stress corresponds to the strength of bent fibre only at extremely low failure probability ($F = 10^{-8}$–10^{-9}). In the more relevant range of failure probability ($F = 10^{-4}$–10^{-5}) increasing the proof stress will not increase static fatigue lifetime in bending applications.

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